

MATH 130A Review: Integration by parts

Facts to Know

Integration by parts

$$\int_a^b u \, dv = u \cdot v \Big|_a^b - \int_a^b v \, du$$

Examples

1. Calculate $\int_0^1 xe^{2x} dx$

$$\begin{aligned} u &= x & du &= dx & = (x) \left(\frac{1}{2} e^{2x} \right) \Big|_0^1 + \int_0^1 \left(\frac{1}{2} e^{2x} \right) dx \\ v &= \frac{1}{2} e^{2x} & dv &= e^{2x} dx & = \frac{1}{2} e^2 - \left[\frac{1}{4} e^{2x} \right]_0^1 &= \frac{1}{2} e^2 - \left(\frac{1}{4} e^{2(1)} - \frac{1}{4} e^{2(0)} \right) \\ &&&& &= \frac{1}{2} e^2 - \frac{1}{4} e^2 + \frac{1}{4} \cdot 1 \\ &&&& &= \frac{1}{4} e^2 + \frac{1}{4} \end{aligned}$$

$$\begin{aligned}
 2. \text{ Calculate } \int_0^1 \frac{(x^2+1)e^{-x}}{u} \frac{du}{dx} dx &= \frac{(x^2+1)(-e^{-x})}{u} \Big|_0^1 - \int_0^1 \frac{(-e^{-x})}{v} \frac{d(x^2+1)}{du} dx \\
 &= (x^2+1)(-e^{-x}) \Big|_0^1 - \underbrace{\int_0^1 \frac{(-2x)e^{-x}}{u} \frac{du}{dx} dx}_{(-2x)(-e^{-x}) \Big|_0^1 - \int_0^1 (-e^{-x}) \frac{d(-2x)}{-2dx} dx} \\
 &\quad \underbrace{\int_0^1 2e^{-x} dx}_{-2e^{-x} \Big|_0^1} \\
 &= (x^2+1)(-e^{-x}) \Big|_0^1 - \left((-2x)(-e^{-x}) \Big|_0^1 - [-2e^{-x}] \Big|_0^1 \right) \\
 &= (-2e^{-1} - 1) - (2e^{-1} - (-2e^{-1} + 2e^0)) \\
 &= -2e^{-1} + 1 - 2e^{-1} + (-2e^{-1} + 2) \\
 3. \text{ Calculate } \int_0^1 \tan^{-1} x dx &= -6e^{-1} + 3
 \end{aligned}$$

$$u = \tan^{-1} x$$

$$\begin{aligned}
 du = 1 dx &= (\tan^{-1} x)(x) \Big|_0^1 - \int_0^1 x \frac{d(\tan^{-1} x)}{dx} dx \\
 &= \frac{\pi}{4} - 0 - \int_0^1 x \frac{1}{1+x^2} dx \\
 &= \frac{\pi}{4} - \left[\frac{1}{2} \ln(1+x^2) \right]_0^1 \\
 &= \frac{\pi}{4} - \frac{1}{2} \ln(2)
 \end{aligned}$$